Fast augmented Lagrangian algorithms for solving convex relaxations of combinatorial optimization problems

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In this talk we discuss three different uses of augmented Lagrangian methods for solving convex relaxations of combinatorial optimization problems.

First, we describe alternating direction augmented Lagrangian (ADAL) methods for minimizing a convex function that is the sum of several functions subject to convex constraints, where each function is relatively easy to minimize separately subject to the constraints. We present both Gauss-Seidel-like and Jacobi-like algorithms that compute an epsilon-optimal solution in $O(1/\epsilon)$ iterations. Nesterov-like accelerated versions that have an $O(1/\sqrt{(\epsilon)})$ iteration complexity are also given. For the case where the sum only involves two functions, our complexity results only require one of the functions to have a Lipschitz continuous gradient. We apply our methods to a varied set of problem classes, including matrix completion, robust principal component analysis (PCA), sparse PCA and sparse inverse covariance for graphical model selection. Some of the problems solved have tens of millions of variables and constraints.

Second, we apply ADAL methods to the the duals of semidefinite relaxations of several combinatorial optimization problems, including frequency assignment, maximum independent set and binary integer quadratic programming problems. In this approach the subproblems that are generated at each iteration of the method have closed form solutions and can be solved efficiently.

Third, we present a a row-by-row (RBR) block coordinate descent method combined with an augmented Lagrangian approach for solving the semidefinite programming relaxations of the max-cut and the maximum independent set problems in graphs. In the RBR method we alternatively fix the $(n-1)$-dimensional principal submatrices of the $n \times n$ positive semidefinite matrix X and use its (generalized) Schur Complement to replace the positive semidefinite constraint by a simple second-order cone constraint. When the RBR method is applied to solve the maxcut SDP relaxation, the optimal solution of the RBR subproblem only involves a single matrix-vector product which leads to a powerful approximate method. To handle linear constraints in Lovász theta function SDP, we combine the RBR method with an augmented Lagrangian function approach.

The results presented in this talk were obtained in collaboration with Bo Huang, Shiqian Ma, Tony Qin, Katya Scheinberg, Wotao Yin and Zaiwen Wen.