Optimization in very large graphs

László Lovász and Katalin Vesztergombi

U. Eötvös Loránd, Hungary

Algorithmic questions concerning very large graphs present novel challenges, and some of the classical algorithmic problems must be rephrased and a new kind of complexity theory emerges. By "very large" we mean a graph whose nodes and edges are too numerous to be treated as a single file of data, and often even the number of nodes is not known. We assume that information about such graphs is obtained by an appropriate sampling procedure.

It is clear that every computational result will only be approximate, but this is not the only problem. What should it mean to compute a perfect matching or a maximum cut in such a graph? One possible approach is to give an algorithm that computes the answer locally, from local data. Making this precise connects optimization with techniques in graph theory developed to handle large graphs (like regularity partitions and graph limits).

There are two in a sense extreme situations when this issue is best understood: dense graphs (in which a positive fraction of edges are present), and graphs with bounded degree. We survey the similarities and differences between these two theories.