Old and new results in discrete tomography

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In this talk we review some results concerning the reconstruction of discrete objects from partial information given by their projections. The reconstruction of a matrix with coefficients in $\{0, 1\}$ is a classical example where the partial information or projections are the number of 1's in each row and in each column. We present necessary and sufficient conditions for the existence of solutions, obtained in the late fifties. We also show how this characterization leads to a polynomial time algorithm for the problem.

The matrix reconstruction problem can be seen as the realization of a bipartite graph with given degrees. This latter problem is an special cases of the more general problem of (reconstruction of) factors in graphs. We review some fundamental results for this problem. The main characteristic being that most variations/generalizations of the problem admit polynomial time algorithms.

The matrix reconstruction problem can also be seen as the reconstruction of a rectangular digital image where each pixel has color white or black. We discuss the generalization of this problem to more than two colors. Unlike what happens in factor problems, most variations of this reconstruction problem are NP-hard.

A third interpretation of the matrix reconstruction problem is the reconstruction of a tiling of a rectangular region, by using white and black square tiles of unit side. We end our presentation by discussing the generalization of this problem to other sets of rectangular tiles. It is known that for almost any set of tiles containing at least one bi-dimensional tile, the reconstruction prob- lem is NP-hard. It is also NP-hard for any set of tiles with more than two tiles. On the other hand, for sets of two unidimensional tiles the complexity is unknown unless the case of two horizontal (resp. vertical) tiles or the case of two dominoes.