Game Theory in Operations Research

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Operations Research techniques have traditionally been applied to problems that involve an organization or agent that optimizes a system centrally. In the last decades, though, it has became increasingly important to incorporate competition to the set of tools available to us to model reality accurately. This arose from the recognition that many of the problems that we routinely solve do not involve just one agent but instead involve many agents that have their own conflicting objectives. The most important tool used to address decision-making in the presence of competition in loosely integrated systems is *Game Theory* and the associated equilibrium concepts.

Some directions of research central to our community where this trend has been significant include transportation networks, telecommunication networks and the Internet, supply chain management, and revenue management, to name a few. Although part of this research makes use of the same methodology and models as those used by economists, a differentiating factor of the Operations Research and Computer Science communities was a focus on computation, which led to the name *Algorithmic Game Theory* (see, e.g., the book 'Algorithmic Game Theory' by Nisan, Roughgarden, Tardos and Vazirani. Cambridge University Press, 2007).

Transportation engineers have been looking at equilibrium concepts since Wardrop postulated in the 1950s that drivers select paths that are shortest under the prevailing traffic conditions. At around the same time, Vickrey proposed to use a pricing mechanism to make commuters internalize the congestion they generate, making self-interested decisions optimal for the system. In the late 1970s, the US Department of Energy developed the PIES energy model that illustrated the practicality of equilibrium models in determining market prices. Some years later, several researchers started to analyze the impact of competition in telecommunication networks. This was accelerated by the recognition that the Internet ceased to be a research network controlled by a few cooperating universities, and became a platform where various for-profit companies compete to make money. This competition is best exemplified by the *net neutrality* debate currently ongoing in many parts of the world whereby governments are deciding what policies (or lack thereof) they will use to regulate the way Internet operates.

Another area that experienced a need to incorporate competition is supply chain management. Although early models dealt with the internal operations of a company in isolation, it has been recognized that the reactions of competitors were an important ingredient of the decision. This led to capacity, inventory and facility location problems that incorporate competition explicitly. Once competition is added to the model, it is not as straightforward to have a system operate optimally because agents are not willing to optimize the welfare of the system, they all focus in their own utility functions. For this reason, a related research trend has been the design of mechanisms and contracts that maximize welfare under competition. Auctions are a very important type of mechanism that has been used extensively. Examples include the allocation of spectrum rights where auctions allowed governments to make hefty revenues compared to previous mechanisms used for the same purpose, placing ads in search engines such as Google, Bing or Yahoo, and deciding who prepares school meals in Chile.

Finally, revenue management was introduced in the 1980s after the deregulation of the airline industry in the US. It consists of a set of procedures designed so airlines can make capacity decisions and allocate fares to sell all the seats in an airplane at the 'right price.' The first to try this was American Airlines defending itself from People's Express who was charging fares that were so low that American Airlines could not effectively compete with it. The idea was so successful that it is now believed that an airline cannot survive without using a methodology of this kind. Although early implementations just computed forecasts of demand at different price levels, it was eventually recognized that it was important for an airline to model the reactions of both competing airlines and those of consumers. These ideas led to modeling consumers as strategic agents that can learn the strategies that sellers design to extract higher revenues. Although many of these tools were originally designed for airlines, currently many other industries that need to price goods or services use them.

In this talk, I will present an overview of the use of Game Theory in Operations Research, focusing on some of the examples described above. The main takeaway is that when there is competition, optimizing a system can be a complex task because the system's manager needs to consider the incentives of agents. To illustrate, it can sometimes happen that an increase in capacity may backfire and degrade the performance for the system's users. A situation like this cannot possibly arise with centralized optimization because a relaxation always leads to better solutions.